Homework (lecture 4):
1, 7, 12, 13, 14, 17, 21, 22, 24, 35, 36, 39, 43, 44, 51, 52
1. The square surface as shown measures 3.2 mm on each side. It is immersed in a uniform electric field with $E = 1800 \text{ N/C}$ and with field lines at an angle of $\theta = 35^\circ$ with a normal to the surface. Calculate the electric flux through the surface.

\[
\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta = (1800 \text{ N/C}) \times (3.2 \times 10^{-3} \text{ m}) \cos(180^\circ - 35^\circ)
\]

\[
\Phi = -1.51 \times 10^{-2} \text{ Nm}^2 / \text{C}
\]
7. A point charge of 1.8 \( \mu C \) is at the center of a cubical Gaussian surface 55 cm on edge. What is the net electric flux through the surface?

Using Gauss's law:

\[
\varepsilon_0 \Phi = q_{\text{enclosed}}
\]

\[
\Phi = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{1.8 \times 10^{-6} \text{C}}{8.85 \times 10^{-12} \text{C}^2 / \text{Nm}^2} = 2.0 \times 10^5 \text{Nm}^2 / \text{C}
\]
14. A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure a shows a cross section. Figure b gives the net flux \( \Phi \) through a Gaussian sphere centered on the particle, as a function of the radius \( r \) of the sphere. (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?

\[ \varepsilon_0 \Phi = q_{\text{enclosed}} \]

(a) For \( r < r_A \) (region 1):

\[ q_{\text{enclosed1}} = q_{\text{particle}} = \varepsilon_0 \Phi_1 \]

\[ q_{\text{particle}} = 8.85 \times 10^{-12} \times 2 \times 10^5 = 1.77 \times 10^{-6} \text{ (C)} \]

\[ \approx 1.8 \text{ (\( \mu \)C)} \]

(b) For \( r_A < r < r_B \) (region 2): \( q_{\text{enclosed2}} = q_{\text{particle}} + q_A = \varepsilon_0 \Phi_2 \)

\[ \Phi_2 = -4 \times 10^5 \text{ (Nm}^2/\text{C)} \] \( \Rightarrow q_A = -5.3 \times 10^{-6} \text{ (C)} \) or \(- 5.3 \text{ \( \mu \)C)}

(c) For \( r_B < r \) (region 3): \( \Phi_3 = 6 \times 10^5 \text{ (Nm}^2/\text{C)} \] \( \Rightarrow q_B \)
17. A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of 8.1 $\mu$C/m$^2$. (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

(a) charge = area $\times$ surface density

\[ q = 4\pi r^2 \sigma = 4 \times 3.14 \times 0.6^2 \times 8.1 \times 10^{-6} = 3.7 \times 10^{-5} \text{ (C)} \]

(b) We choose a Gaussian surface covers whole the sphere, using Gauss' law:

\[ \Phi = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{3.7 \times 10^{-5}}{8.85 \times 10^{-12}} = 4.2 \times 10^6 \text{ Nm}^2 / \text{C} \]
21. An isolated conductor of arbitrary shape has a net charge of $+10 \times 10^{-6}$ C. Inside the conductor is a cavity within which is a point charge $q = +3.0 \times 10^{-6}$ C. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?

(a) Consider a Gaussian surface within the conductor that covers the cavity wall, in the conductor, $E = 0$:

$$ q_{\text{wall}} + q_{\text{point}} = 0 $$

$$ q_{\text{wall}} = -q_{\text{point}} = -3 \times 10^{-6} \text{ C or } -3 \mu \text{C} $$

(b) the total charge of the conductor:

$$ q_{\text{wall}} + q_{\text{outer}} = 10 \times 10^{-6} \Rightarrow q_{\text{outer}} = 13 \times 10^{-6} \text{ C or } 13 \mu \text{C} $$
22. An electron is released from rest at a perpendicular distance of 9 cm from a line of charge on a very long nonconducting rod. That charge is uniformly distributed, with 4.5 $\mu$C per meter. What is the magnitude of the electron’s initial acceleration?

**Electric field at point P:**

\[ E = \frac{\lambda}{2\pi\varepsilon_0 R} \]

**Force acting on the electron:**

\[ F = eE = \frac{e\lambda}{2\pi\varepsilon_0 R} = ma \Rightarrow a = \frac{e\lambda}{2\pi\varepsilon_0 mR} \]

\( R = 9 \text{ cm} = 0.09 \text{ m} \)
\( \lambda = 4.5 \mu \text{C/m} = 4.5 \times 10^{-6} \text{ C/m} \)
36. The figure shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density \( \sigma = 2.31 \times 10^{-22} \) C/m\(^2\). In unit-vector notation, what is \( \vec{E} \) at points (a) above the sheets, (b) between them, and (c) below them?

For one non-conducting sheet:

\[
E = \frac{\sigma}{2\varepsilon_0}
\]

Using the superposition to calculate \( E \) due to two sheets:

(a) \[
E = 2 \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{2.31 \times 10^{-22}}{8.85 \times 10^{-12}} = 2.61 \times 10^{-11} (N/C)
\]

The net electric field direction is upward \( \vec{E} = 2.61 \times 10^{-11} (N/C) \hat{j} \)

(b) \( E = 0 \)

(c) \( \vec{E} = -2.61 \times 10^{-11} (N/C) \hat{j} \)

The direction is downward
39. A small, nonconducting ball of mass $m = 1 \text{ mg}$ and charge $q = 2 \times 10^{-8} \text{ C}$ hangs from an insulating thread that makes an angle $\theta = 30^0$ with a vertical, uniformly charged nonconducting sheet. Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density $\sigma$ of the sheet.

If the ball is in equilibrium:

$$\vec{F} + \vec{F}_g + \vec{T} = 0$$

$$\tan \theta = \frac{F}{F_g} = \frac{qE}{mg} = \frac{q}{mg} \times \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = \frac{2\varepsilon_0 mg \tan \theta}{q} = 5 \times 10^{-9} (\text{C/m}^2)$$
44. The figure gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. What is the charge on the sphere?

\[ E = \left( \frac{q}{4\pi\varepsilon_0 R^3} \right) r \quad (r \leq R) \]

- At \( r = 2 \text{ cm} \), \( E \) is maximum, so \( R = 2 \text{ cm} \)

\[
E = \frac{q}{4\pi\varepsilon_0 R^2}
\]

\[
q = 4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.02)^2 \times (5 \times 10^7)
\]

\[
= 2.2 \times 10^{-6} (C)
\]
51. A nonconducting spherical shell of inner radius \( a = 2 \text{ cm} \) and outer radius \( b = 2.4 \text{ cm} \) has a positive volume charge density \( \rho = \frac{A}{r} \), where \( A \) is a constant and \( r \) is the distance from the center of the shell. In addition, a small ball of charge \( q = 45 \text{ fC} \) is located at that center. What value should \( A \) have if the electric field in the shell \( (a \leq r \leq b) \) is to be uniform? 

**Key idea:** First, we need to calculate \( E \) inside the shell, if the field is uniform, so \( E \) is independent of distance from the center

\[
E_r = \frac{1}{4\pi\varepsilon_0} \frac{Q_{\text{total}}}{r^2}
\]

\( Q_{\text{total}} = q + Q_{\text{shell}} \)

\( Q_{\text{shell}} \) is the enclosed charge in the shell of thickness \( r_b - a \):

\[
dQ_{\text{shell}} = \rho \times dV = \rho \times 4\pi r^2 dr
\]

\[
Q_{\text{shell}} = 4\pi \int_{a}^{b} \frac{A}{r} r^2 dr = 2\pi A (r_b^2 - a^2)
\]
\[ dV = 4\pi r^2 dr \]
\[ dq = \rho \times dV \]
Using Gauss' law:

\[ \varepsilon_0 \Phi = q_{\text{total}} \]

\[ \varepsilon_0 E 4\pi r_G^2 = q_{\text{total}} \]

\[ E = \frac{q_{\text{total}}}{\varepsilon_0 4\pi r_G^2} = \frac{q + 2\pi A (r_G^2 - a^2)}{4\pi \varepsilon_0 r_G^2} \]

We rewrite:

\[ E = \frac{A}{2\varepsilon_0} + \frac{1}{2\varepsilon_0} \left( \frac{q}{2\pi} - Aa^2 \right) \times \frac{1}{r_G^2} \]

If \( E \) is uniform in the shell:

\[ \frac{q}{2\pi} - Aa^2 = 0 \Rightarrow A = \frac{q}{2\pi a^2} \]

\[ A = \frac{45 \times 10^{-15} \, C}{2 \times 3.14 \times (0.02 \, m)^2} = 1.79 \times 10^{-11} \, (C/\, m^2) \]
52. The figure below shows a spherical shell with uniform volume charge density \( \rho = 1.56 \text{ nC/m}^3 \), inner radius \( a = 10 \text{ cm} \), and outer radius \( b = 2a \). What is the magnitude of the electric field at radial distances (a) \( r = 0 \), (b) \( r = a/2 \), (c) \( r = a \), (d) \( r = 1.5a \), (e) \( r = b \), and (f) \( r = 3b \)?

For (a), (b), (c) using Gauss's law, we find \( E = 0 \). For (d), (e) \( a \leq r \leq b \):

The enclosed charge:

\[
q_{enc} = \rho \times V = \rho \left( \frac{4}{3} \pi r^3 - \frac{4}{3} \pi a^3 \right)
\]

The electric field:

\[
E = \frac{1}{4 \pi \varepsilon_0} \frac{q_{enc}}{r^2} = \frac{1}{4 \pi \varepsilon_0} \frac{\rho \times \frac{4}{3} \pi (b^3 - a^3)}{r^2}
\]

For (f):

\[
E = \frac{1}{4 \pi \varepsilon_0} \frac{q_{total}}{r^2} = \frac{1}{4 \pi \varepsilon_0} \frac{\rho \times \frac{4}{3} \pi (b^3 - a^3)}{r^2} = \frac{\rho}{3 \varepsilon_0} \frac{b^3 - a^3}{r^2}
\]
Homework (lecture 5):
1, 6, 8, 11, 14, 18, 24, 28, 29, 35, 43, 59, 60, 64
1. A particular 12 V car battery can send a total charge of 84 A.h through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

(a) In the previous lecture, we mentioned that the coulomb unit is derived from ampere for electric current $i$:

$$i = \frac{dq}{dt} \implies dq = idt$$

$$Q = 84 \left( \frac{C}{s} \right) \times 3600 \left( s \right) = 3 \times 10^5 \left( C \right)$$

(b) Energy is computed by:

$$\Delta U = \Delta V \times Q = 12 \times 3 \times 10^5 = 3.6 \times 10^6 \left( J \right)$$
6. When an electron moves from A to B along an electric field, see the figure. The electric field does $4.78 \times 10^{-19} \text{ J}$ of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?

(a) We have work done by the electric field:

$$W = -q\Delta V$$

$$W = -(-e)(V_B - V_A)$$

$$V_B - V_A = \frac{W}{e} = \frac{4.78 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.0(V)$$

(b) $$V_C - V_A = V_B - V_A = 3.0(V)$$

(c) $$V_C - V_B = 0$$: on the same equipotential
18. Two charged particles are shown in Figure a. Particle 1, with charge $q_1$, is fixed in place at distance $d$. Particle 2, with charge $q_2$, can be moved along the $x$ axis. Figure b gives the net electric potential $V$ at the origin due to the two particles as a function of the $x$ coordinate of particle 2. The plot has an asymptote of $V = 5.92 \times 10^{-7} \text{ V}$ as $x \to \infty$. What is $q_2$ in terms of $e$?

Potential due to a point charge: \[ V = k \frac{q}{r} \]

Potential at the origin (O) due to $q_1$ and $q_2$:

\[ V_O = k \frac{q_1}{d} + k \frac{q_2}{x} \]

\[ V_{O,x=\infty} = k \frac{q_1}{d} = 5.92 \times 10^{-7} \text{ (V)} \]

At $x = 8 \text{ cm}$, $V_O = 0$:

\[ V_{O,x=8} = V_{O,x=\infty} + k \frac{q_2}{x} \]

\[ q_2 = -\frac{V_{O,x=\infty}x}{k} = -\frac{5.92 \times 10^{-7} \times 0.08}{8.99 \times 10^9} = -5.27 \times 10^{-18} \text{ (C) or } -33e \]
24. The figure shows a plastic rod having a uniformly distributed charge \( Q = -28.9 \text{ pC} \) has been bent into a circular arc of radius \( R = 3.71 \text{ cm} \) and central angle \( \Phi = 120^0 \). With \( V=0 \) at infinity, what is the electric potential at \( P \), the center of curvature of the rod?

Consider potential at \( P \) due to an element \( dq \):

\[
dV = k \frac{dq}{R}
\]

\[
V = \int k \frac{dq}{R} = k \frac{Q}{R}
\]

\[
V = \frac{8.99 \times 10^9 \times (-28.9 \times 10^{-12})}{3.71 \times 10^{-2}} = -7.0(V)
\]
35. The electric potential at points in an xy plane is given by

\[ V = (2 \text{ V/m}^2)x^2 - (3 \text{ V/m}^2)y^2 \]

In unit vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

We have:

\[ \vec{E} = -\vec{\nabla}V \]

\[ \begin{align*}
E_x &= -\frac{\partial V}{\partial x} = -\frac{\partial V}{\partial x} = -4x = -12(V/m) \\
E_y &= -\frac{\partial V}{\partial y} = -\frac{\partial V}{\partial y} = 6y = 12(V/m)
\end{align*} \]

\[ \vec{E} = -12(V/m)\hat{i} + 12(V/m)\hat{j} \]
43. How much work is required to set up the arrangement of the figure below if \( q = 2.3 \text{ pC} \), \( a = 64 \text{ cm} \), and the particles are initially infinitely far apart and at rest?

We have 4 charges, so we have \( N = 6 \) pairs:

\[
N = \frac{n(n - 1)}{2}
\]

\[
W_{\text{applied}} = U_{\text{system}}
\]

\[
U_{\text{system}} = kq^2 \left( -\frac{1}{a} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a} \right)
\]

\[
U_{\text{system}} = \frac{2kq^2}{a} \left( \frac{1}{\sqrt{2}} - 2 \right)
\]

Note: \( q = 2.3 \text{ pC} = 2.3 \times 10^{-12} \text{ C} \); \( a = 64 \text{ cm} = 0.64 \text{ m} \)
64. A hollow metal sphere has a potential of $+300$ V with respect to ground (defined to be at $V = 0$) and a charge of $5.0 \times 10^{-9}$ C. Find the electric potential at the center of the sphere.

$V = \text{constant} = +300$ V throughout the entire conductor, this is valid for solid and hollow metal spheres.
Homework (lecture 6):
2, 4, 6, 11, 14, 16, 26, 31, 33, 42, 48, 51
2. The capacitor in the figure below has a capacitance of 30 \( \mu \text{F} \) and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

S is closed, the charge on the capacitor plates is:

\[ q = CV \]

\[ q = 30 \times 10^{-6} \times 120 = 3.6 \times 10^{-3} \text{ (C)} \]
11. In the figure below, find the equivalent capacitance of the combination. Assume that $C_1 = 10.0 \ \mu F$, $C_2 = 5.0 \ \mu F$, and $C_3 = 4.0 \ \mu F$

$C_1$ and $C_2$ are in parallel, the equivalent capacitance:

$$C_{12} = C_1 + C_2 = 15(\mu F)$$

$C_{12}$ and $C_3$ in series:

$$C_{123} = \frac{C_{12}C_3}{C_{12} + C_3} = \frac{15 \times 4}{15 + 4} = 3.16(\mu F)$$
16. Plot 1 in Figure a gives the charge \( q \) that can be stored on capacitor 1 versus the electric potential \( V \) set up across it. Plots 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure b shows a circuit with those three capacitors and a 10.0 V battery. What is the charge stored on capacitor 2 in that circuit?

\[
C_1 = \frac{q_1}{V_1} = \frac{12(\mu C)}{2(V)} = 6\mu F; \quad C_2 = \frac{q_2}{V_2} = \frac{8(\mu C)}{2(V)} = 4\mu F; \quad C_3 = \frac{q_3}{V_3} = \frac{4(\mu C)}{2(V)} = 2\mu F
\]

\[
C_{123} = 3(\mu F)
\]

\[
V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{1}{2} \times 10 = 5(V) \quad \Rightarrow \quad q_2 = C_2V_2 = 4\mu F \times 5V = 20\mu C
\]
26. Capacitor 3 in Figure a is a variable capacitor (its capacitance $C_3$ can be varied). Figure b gives the electric potential $V_1$ across capacitor 1 versus $C_3$. Electric potential $V_1$ approaches an asymptote of 8 V as $C_3 \rightarrow \infty$. What are (a) the electric potential $V$ across the battery, (b) $C_1$, and (c) $C_2$?

(a) When $C_3 \rightarrow \infty$, $C_{123} = C_1$; so, $V = V_1 = 8$ V
(b) 

\[
C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{C_1 (C_2 + C_3)}{C_1 + C_2 + C_3};
\]
\[ V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{C_2 + C_3}{C_1 + C_2 + C_3} V \]

- At \( C_3 = 0 \), \( V_1 = 2 \text{ V} \):
  \[ C_1 = 3C_2 \]

- At \( C_3 = 6 \ \mu\text{F} \), \( V_1 = 5 \text{ V} \):
  \[ V_1 = \frac{C_2 + 6}{3C_2 + C_2 + 6} \quad 8 = 5 \]

\[ C_2 = 1.5 \mu\text{F}; \ C_1 = 4.5 \mu\text{F} \]
33. A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to \( V = 0 \) at infinity. Calculate the energy density in the electric field near the surface of the sphere.

In a general case, the energy density is computed by:

\[
u = \frac{1}{2} \varepsilon_0 E^2\]

For a charged isolated metal sphere:

\[
u = \frac{1}{2} \varepsilon_0 \left(\frac{V}{R}\right)^2 = \frac{1}{2} 8.85 \times 10^{-12} \left(\frac{8000}{0.05}\right)^2 = 0.113(\text{J/m}^3)\]
42. A parallel-plate air-filled capacitor has a capacitance of 50 pF: (a) If each of its plates has an area of 0.30 m², what is the separation? (b) If the region between the plates is now filled with material having \( \kappa = 5.6 \), what is the capacitance?

(a) For parallel-plate capacitors:

\[
C = \frac{\varepsilon_0 A}{d}
\]

\[
d = \frac{\varepsilon_0 A}{C} = \frac{8.85 \times 10^{-12} \times 0.30}{50 \times 10^{-12}} = 5.3 \times 10^{-2} (m) = 5.3 (cm)
\]

(b) With a dielectric:

\[
C' = \kappa C = 5.6 \times 50 = 280 (pF)
\]
48. The figure below shows a parallel-plate capacitor with a plate area \( A = 5.56 \text{ cm}^2 \) and separation \( d = 5.56 \text{ mm} \). The left half of the gap is filled with material of dielectric constant \( \kappa_1 = 7.00 \); the right half is filled with material of dielectric constant \( \kappa_2 = 12.0 \). What is the capacitance?

Their configuration is equivalent to a combination of two capacitors in parallel with dielectrics \( \kappa_1 \) and \( \kappa_2 \), respectively.

\[
C_0 = \frac{\varepsilon_0 (A/2)}{d} = \frac{8.85 \times 10^{-12} \times 5.56 \times 10^{-4}}{2 \times 5.56 \times 10^{-3}} = 4.43 \times 10^{-13} \text{ (F)}
\]

\[
C_0 = 0.443 \text{ pF}
\]

\[
C_{\text{equivalent}} = C_1 + C_2 = (\kappa_1 + \kappa_2)C_0 = 8.42 \text{ (pF)}
\]